Canonical covariant formalism for Dirac-Nambu-Goto bosonic p-branes and the Gauss-Bonnet topological term in string theory

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Abstract

Using a covariant and gauge invariant geometric structure constructed on the Witten covariant phase space for Dirac-Nambu-Goto bosonic p-branes propagating in a curved background, we find the canonically conjugate variables, and the relevant commutation relations are considered, as well as, we find the canonical variables for the Gauss-Bonnet topological term in string theory.

I. INTRODUCTION

Relativistic extended objects such as strings and membranes, from the physical point of view can be considered as fundamental building blocks of field theories, and represent the more viable candidates for the quantum theory of gravity [1, 2]. On the mathematical side, string and membranes have been used to motivate unsuspected interplay among some mathematical subjects. For example, at the perturbative level, it is well known that string theory is related to the theory of Riemann surfaces [3] and some aspects of algebraic geometry and Mirror symmetry.

The quantization of such objects is a very complicated problem in physics because, among other things, the theory is highly non-lineal and the standard methods cannot be applied directly, even for extended objects of simple topologies. However, in recent works [4, 5], using a covariant description of the canonical formalism for quantization [6], and the deformations formalism introduced by Capovilla-Guven [CG] in [7], the basis for the study of the symmetries and the quantization aspects of Dirac-Nambu-Goto bosonic p-branes [DNG]

propagating in a curved background space-time have been developed. Such basis consist in the construction a covariant and gauge invariant symplectic structure on the corresponding quotient phase space Z (the space of solutions of classical equations of motion divided by symmetry group volume) instead of choosing a special coordinate system on the phase space, with coordinates p_i and q^i as we usually find in the literature. The crucial observation of this work is that such a choice is not necessary.

However, is remarkable to mention that recently there are works where using the constrained Hamiltonian approach [8] have been keeping the reparametrization symmetry intact [9, 10, 11, 12] , we will discuss this approach in the appendix part with the results of this paper.

In this manner, the purpose of this article is to consider the results obtained previously in [4, 5] and using basic ideas of symplectic geometry in order to find; the canonically conjugate variables, the corresponding Poincaré charges, and the fundamental Poisson brackets-commutators in a covariant description, which is absent in the literature, as well as, to find the canonical variables for Gauss-Bonnet term [GB] in string theory as first step to find a possible contribution due to such term at quantum level, just as it was commented in [17].

This paper is organized as follows. In Sect.II, using the results presented in [4, 5] and standard ideas of symplectic geometry, we identify the covariant canonically conjugate variables that we will use in the development of this paper. In Sect.III, with the results found in the last section, we obtain the Poincaré charges and its corresponding laws of conservation, confirming the results presented in [13], as well as the Poisson bracket and the Poincaré algebra also are discussed. In Sect. IV using the results given in [17] we find the canonical variables for [GB] topological term using an alternative method that we introduced in previous sections. In Sect. V we give some remarks and prospects.

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II. Symplectic geometry in Dirac-Nambu-Goto bosonic p-branes

In [4, 5] a covariant and gauge invariant symplectic structure for [DNG] bosonic p-branes propagating in a curved background has been constructed, and is given by

$$\omega = \alpha \int_{\Sigma} \delta(-\sqrt{-\gamma} e^{a}{}_{\mu} \delta X^{\mu}) d\Sigma_a, \tag{1}$$

where $e^{a}{}_{\mu}$ is a vector field tangent to the world-volume created by the extendon, δX^{μ} is an infinitesimal space-time variation of the embedding [7], and δ, the deformation operator that acts as exterior derivative on the phase space [4, 5]. It is important to mention that ω turns out to be independent on the choice of Σ,*i.e.*, $\omega_{\Sigma} = \omega_{\Sigma'}$ where Σ is a Cauchy p-surface, and it will be a very important property of ω, since it

allows us to establish a connection between functions and Hamiltonian vector fields on Z; this subject will be considered in the next paragraphs.

Because of $d\Sigma_a = \tau_a d\Sigma$, being τ_a a normalized ($\tau_a \tau^a = -1$) timelike vector field tangent to the world-volume, ω can be rewritten as

$$\omega = \int_\Sigma \delta \hat{p_\mu} \wedge \delta X^\mu d\Sigma, \tag{2}$$

where, $\hat{p_\mu} = -\sqrt{-\gamma} p_\mu$, and $p_\mu = -\alpha e^a{}_\mu \tau_a$, satisfying

$$p_\mu p^\mu = -\alpha^2, \tag{3}$$

which is the mass shell condition for the p-brane. We can see that p_μ is proportional to the timelike unit normal to Σ into the world-volume.

Since the formalism of deformations introduced in [7] is weakly covariant, the embedding functions depend of local coordinates for the world-volume (ξ^a), and we have that $\hat{p_\mu} = \hat{p_\mu}(\overrightarrow{\xi}, \tau)$, and $X^\mu = X^\mu(\overrightarrow{\xi}, \tau)$, where we split the local coordinates for the world-volume in an arbitrary evolution parameter τ and coordinates $\overrightarrow{\xi}$ for Σ at fixed values of τ. In this manner from equation (2) we can identify $\hat{p_\mu}(\overrightarrow{\xi}, \tau)$ as the canonical conjugate momentum to the embedding function $X^\mu(\overrightarrow{\xi}, \tau)$ for [DNG] p-branes in a curved background. Therefore, any function f on the covariant phase space depends of $\hat{p_\mu}$ and X^μ, $f = f(\hat{p_\mu}, X^\mu)$.

Now, using basic ideas of symplectic geometry [9-11], we know that if the symplectic structure ω of the theory under study is invariant under a group of transformations G, (in our case ω is invariant under space-time diffeomorphisms) which corresponds to the gauge transformations of [DNG] theory [4, 5], the Lie derivative along a vector V tangent to a gauge orbit of G of ω vanishes, that is,

$$\pounds_V \omega = V \rfloor \delta\omega + \delta(V \rfloor \omega) = 0, \tag{4}$$

where $\rfloor$ denotes the operation of contraction with V. Since ω is an exact and in particular closed two-form, $\delta\omega = 0$ [4, 5], and we have that Eq. (4), at least locally, takes the form

$$V \rfloor \omega = -\delta H, \tag{5}$$

where H is a function on Z which we call the generator of the G transformations [6]. In this manner, the relation (5) allows us to establish a connection between functions and Hamiltonian vector fields on Z, just as we commented previously.

On the other hand, if h and g are functions on the phase space, we can define using the symplectic structure ω a new function $[f, g]$, the Poisson bracket of h and g, as

$$[h, g] = V_h \rfloor g = -V_g \rfloor h, \tag{6}$$

where V_h and V_g correspond to the Hamiltonian vector fields generated by h, and g respectively through Eq. (4).

With these results, in the next section we will calculate in a weakly covariant way the relevant Poisson bracket, the Poincaré charges and their respective conservation laws; we show also the closeness of the Poincaré algebra.

III. The Poisson bracket, Poincaré charges and Poincaré algebra in a flat space-time

For our aims, first we will find the vector fields associated with the fundamental canonical variables on Z, $(\hat{p_\mu}, X^\mu)$. For this, we use the expression (5), finding that

$$\begin{array}{ccc} X^\alpha & \longrightarrow & V_{X^\alpha} = -\dfrac{\partial}{\partial p_\alpha} \\ \hat{p_\alpha} & \longrightarrow & V_{p_\alpha} = \dfrac{\partial}{\partial X^\alpha}, \end{array} \tag{7}$$

in this manner, using the equation (6) we have,

$$[X^\mu(\overrightarrow{\xi}', \tau), X^\nu(\overrightarrow{\xi}, \tau)] = [\hat{p_\mu}(\overrightarrow{\xi}', \tau), \hat{p_\nu}(\overrightarrow{\xi}, \tau)] = 0, \tag{8}$$

$$[X^\mu(\overrightarrow{\xi}, \tau), \hat{p_\nu}(\overrightarrow{\xi}', \tau)] = \delta^\mu{}_\nu \delta(\overrightarrow{\xi} - \overrightarrow{\xi}'), \tag{9}$$

where $\delta^\mu{}_\nu$ is the Kronecker symbol and $\delta(\overrightarrow{\xi} - \overrightarrow{\xi}')$ the Dirac delta function. In Eqs. (8) and (9) we note that the parameter τ is a world-surface coordinate, this implies a coordinate gauge fixing condition, because of the invariance of the theory under world-surface parametrizations, contrary to the results found in [16] that use a strongly covariant formalism.

On the other hand, if in Eq. (5) we choose $V = \epsilon^\alpha \frac{\partial}{\partial X^\alpha}$, where ϵ^α is a constant space-time vector, we find

$$V \rfloor \omega = -\delta[-\epsilon^\mu \tau_a (\alpha \sqrt{-\gamma} e^a{}_\mu)], \tag{10}$$

where we can identify the linear momentum density

$$P^{a\mu} = -\alpha \sqrt{-\gamma} e^{a\mu}, \tag{11}$$

that is tangent to the world-volume and parallel to the tangent vector $e^{a\mu}$. Furthermore, using the Gauss-Weingarten equations [7]

$$\begin{aligned} \nabla_a e^\mu{}_b &= -K_{ab}{}^i n_i{}^\mu, \\ \widetilde{\nabla}_a n_i{}^\mu &= K_{abi} e^{\mu b}, \end{aligned} \tag{12}$$

and remembering that the solutions to the equations of motion for [DNG] p-branes corresponds to extremal surfaces ($K^i = 0$) [5], we find that the linear momentum are covariantly conserved, this is

$$\nabla_a P^{a\mu} = 0. \tag{13}$$

On the other hand, we can express the total linear momentum P^μ as

$$P^\mu = \int_\Sigma P^{a\mu} d\Sigma_a = \int_\Sigma \hat{p^\mu} d\Sigma, \tag{14}$$

where we can see that the total linear momentum, equation (14), corresponds to the canonical momentum $\hat{p^\mu}$ given in equation (2). Then the total lineal momentum and the canonical momentum coincide.

Now, we will find the angular momentum of the extendon. For this we rewrite our symplectic structure given in equation (1) as [5]

$$\omega = \int_\Sigma \delta(\sqrt{-\gamma} e^a{}_\mu \delta X^\mu) d\Sigma_a = \int_\Sigma \sqrt{-\gamma} n_{i\alpha} \delta X^\alpha \wedge \widetilde{\nabla}^a (n^i{}_\mu \delta X^\mu) d\Sigma_a, \tag{15}$$

where the $n^i{}_\mu$ are vector fields normal to the world-volume. In this manner, for a vector field given by $V = \frac{1}{2}[a_\alpha{}^\beta X^\alpha \frac{\partial}{\partial X^\beta} - a^\alpha{}_\beta X^\beta \frac{\partial}{\partial X^\alpha}]$, with $a_{\alpha\beta} = -a_{\beta\alpha}$, the contraction $V\rfloor\omega$, with ω expressed as in Eq. (15), gives

$$\begin{aligned} V\rfloor\omega &= \sqrt{-\gamma}\frac{1}{2} a_{\alpha\beta}[n_i{}^\beta X^\alpha \widetilde{\nabla}^a \phi^i - \phi_i n^{i\beta} e^{a\alpha} - \phi_i X^\alpha \widetilde{\nabla}^a n^{i\beta} - n_i{}^\alpha X^\beta \widetilde{\nabla}^a \phi^i + \phi_i X^\beta \widetilde{\nabla}^a n^{i\alpha} + \phi_i e^{a\beta} n^{i\alpha}] \\ &= \sqrt{-\gamma}\frac{1}{2} a_{\alpha\beta}[X^\alpha D_\delta e^{a\beta} - \phi_i n^{i\beta} e^{a\alpha} - X^\beta D_\delta e^{a\alpha} + \phi_i e^{a\beta} n^{i\alpha}] \\ &= \delta(a_{\alpha\beta}\frac{1}{2}[P^{a\beta} X^\alpha - P^{a\alpha} X^\beta]), \end{aligned} \tag{16}$$

where we have used the equation (12) and $D_\delta e_a = K_{ab}{}^i \phi_i e^b + \widetilde{\nabla}_a \phi_i n^i$ [7]; thus, from the last equation we can identify the angular momentum of the p-brane given by

$$M^{a\beta\alpha} = \frac{1}{2}[P^{a\beta} X^\alpha - P^{a\alpha} X^\beta], \tag{17}$$

and using equation (12) and the equation of motion ($K^i = 0$), we get that

$$\nabla_a M^{a\beta\alpha} = 0, \tag{18}$$

this is, the angular momentum is covariantly conserved. It is important to mention that the laws of conservation given by equations (13) and (18) correspond exactly to those found in [13], but obtained in a different way.

We define the total angular momentum $M^{\alpha\beta}$ as

$$M^{\alpha\beta} = \int_\Sigma M^{\alpha\beta} d\Sigma_a = \int_\Sigma (\hat{p^\beta} X^\alpha - \hat{p^\alpha} X^\beta) d\Sigma. \tag{19}$$

In this manner, using equation (19) we can find the Hamiltonian vector field associated to the angular momentum, using Eq. (5) with $H^{\alpha\beta} = (\hat{p^\beta} X^\alpha - \hat{p^\alpha} X^\beta)$ we find

$$V^{\beta\alpha} = \eta^{\lambda\beta}\left(X^\alpha \frac{\partial}{\partial X^\lambda} + \hat{p^\alpha}\frac{\partial}{\partial \hat{p^\lambda}}\right) - \eta^{\lambda\alpha}\left(X^\beta \frac{\partial}{\partial X^\lambda} + \hat{p^\beta}\frac{\partial}{\partial \hat{p^\lambda}}\right), \tag{20}$$

where $\eta^{\mu\nu}$ is the Minkowski background metric. Thus, using the equation (20) we can calculate the brackets $[M^{\mu\nu}, M^{\alpha\beta}]$ and $[M^{\mu\nu}, P^\alpha]$, finding

$$\begin{aligned} [M^{\mu\nu}, M^{\alpha\beta}] &= \eta^{\nu\alpha} M^{\mu\beta} + \eta^{\mu\alpha} M^{\beta\nu} + \eta^{\nu\beta} M^{\alpha\mu} + \eta^{\mu\beta} M^{\nu\alpha}, \\ [M^{\mu\nu}, P^\alpha] &= \eta^{\mu\alpha} P^\nu - \eta^{\alpha\nu} P^\mu, \end{aligned} \tag{21}$$

therefore, we can see that the Poincaré charges P and M indeed close correctly on the Poincaré algebra, such as in the case of string theory using a standard canonical formalism [1].

IV. The canonical variables for The Gauss-Bonnet topological term in string theory

Along the same lines, in this section using the covariant canonical formalism we will find the canonical variables for the [GB] topological term in string theory. It is not possible to obtain this results usig the conventional canonical formalism because we do not find any contribution to the equations of motion, the reason being that the field equations of the [GB] topological term are proportional to the called Einstein tensor, and it does not give any contribution to the dynamics in a two-dimension worldsheet swept out by a string, since the Einstein tensor vanishes for such a geometry [17]. However, we identify from a nontrivial covariant and gauge invariant symplectic structure constructed in [17] the canonical variables for such term. It is difficult to find the canonical variables for [GB] topological term as in the last section for [DNG] p-branes, because the symplectic structure for [GB] is not trivial [17], therefore, we need to use an alternative method for such purposes. This method consists in exploiting the frame gauge dependence of the connection coefficients ${\gamma_{cd}}^a$ (which means that it can always be set equal to zero at any single point chosen by an appropriate choice of the relevant frames) and the integral kernel of the geometric structure for [GB] in string theory is given in terms of deformations of such coefficients [17], and to rewrite the connection in terms of the rotation (co)vector such as in [19] which will be crucial for our developments.

For our aims, we need the covariant and gauge invariant symplectic structure constructed in [17] given by

$$\omega = \int_\Sigma \delta\Psi^a d\Sigma_a, \tag{22}$$

where Ψ^a is identified as a symplectic potential for [GB] in string theory

$$\Psi^a = \sqrt{-\gamma}[\beta \gamma^{cd} \delta \gamma_{cd}{}^a - \beta \gamma^{ab} \delta \gamma_{cb}{}^c], \tag{23}$$

here, β is a constant, $\gamma_{cd}{}^a$ are the connection coefficients [7], and δ is identified as exterior derivative on the phase space [17].

On the other hand, we know that for the case of string theory the embedding 2-surface is characterized by an antisymmetric unit tangent element tensor given by $\varepsilon^{\mu\nu} = \varepsilon^{AB} l_A{}^\mu l_B{}^\nu$ (where ε^{AB} are the constant components of the standard two-dimensional flat space alternating tensor) [18], thus, in terms of the deformations formalism given in [7] we can introduce the rotation (co)vector ρ_a given in terms of the frame gauge coefficients connection $\gamma_{ab}{}^c$ and ε^{ab}

$$\rho_b = \gamma_{bd}{}^c \varepsilon^d{}_c, \tag{24}$$

and this implies that

$$\gamma_{bd}{}^c = \frac{1}{2} \varepsilon^c{}_d \rho_b, \tag{25}$$

we can note that the frame gauge dependence of $\gamma_{ab}{}^c$ induces the same gauge dependence on ρ_a.

Considering equations (24), (25) and the gauge dependence on ρ_b, we can prove that Ψ^a given by Eq. (23) takes the form

$$\Psi^a = \sqrt{-\gamma} \varepsilon^{ab} \rho_b, \tag{26}$$

thus, we can write the symplectic structure given in Eq. (22) as

$$\omega = \int_\Sigma \delta(\sqrt{-\gamma} \varepsilon^{ab} \tau_a) \wedge \delta(\rho_b) d\Sigma, \tag{27}$$

this last equation has the same form of Eq. (2) for [DNG] p-branes, in this manner we can identify $p^a = \sqrt{-\gamma} \varepsilon^{ab} \tau_a$ and $q_b = \rho_b$ as canonical variables for the [GB] topological term. It is remarkable to mention that these canonical variables have indices of the worldsheet contrary to the canonical variables for [DNG] p-branes that has indices of space-time (see Eq. (2)). This fact is important because if we add the [GB] topological term to [DNG] action in string theory and using the deformations formalism that we used here to identify the canonical variables for this system is very difficult, because combine variables wiht spacetime and worldsheet indices. However, this problem can be clarified using the ideas presented in [19] where a strongly covariant formalism is used, and in this case we can find the canonical variables for [GB] topological term with spacetime indices and perhaps could be more easy to work the system [GB] and [DNG] in string theory [20].

V. Conclusions and prospects

As we can see, identification of the covariant canonical variables for [DNG] p-branes is easy because the expression of the symplectic structure for the theory under study is simple. In this manner, we could find the Poincaré charges, conservation laws in a different way presented in [13], and construct the relevant Poisson brackets. With these results we have the elements to study the quantization aspects in a covariant way and in particular case of string theory which is absent in the literature.

It is important to emphasize that the canonical variables for [GB] found in this paper has worldsheet indices, and if we add the [GB] term in any action describing strings, for example [DNG] system, we need to identify the pullback on the canonical variables found of this paper to obtain the canonical variables in terms of spacetime indices, however, using the strongly covariant formalism used in [19] this problem can be clarified. The only problem is to find a canonical transformation that leaves the symplectic structure in the Darboux form with some new variables, P and Q, say, that contained the canonical variables of [DNG] and [GB] systems, and thus, we will see the contribution to quantum level of the [GB] term when we add it the [DNG] action in string theory, however, this we will discuss in future works (partial results is given in [20]).

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Appendix

In this section, we will try to stablish a conection with the results given in [12] taking a particular case of the results found in this paper. For our purposes, we will consider a relativistic extended object Σ, of dimension p, embedded in arbitrary fixed $(N+1)$- dimensional background spacetime $\{M, g_{\mu\nu}\}$, Σ is described locally by the spacelike embedding $x^\mu = X^\mu(u^A)$, where x^μ are local coordinates for the back-

ground spacetime, u^A local coordinates for Σ, and X^μ the embedding functions ($\mu, \nu, ..., = 0, 1, ..., N$, and $A, B, ..., = 1, ..., p$).

The tangent vectors to Σ are defined by $\epsilon^\mu_A = \partial_A X^\mu \partial_\mu$, so that the positive definite metric on Σ is

$$h_{AB} = g(\epsilon_A, \epsilon_B) = \epsilon^\mu_A \epsilon^\nu_B g_{\mu\nu}, \tag{28}$$

note that we can construct out of the metric h_{AB} the intrinsic geometry of Σ (for more details about the intrinsic and extrinsic geometry see [5, 7]).

We consider now the time evolution of Σ in spacetime. We denote its trajectory, or worldvolume, by m. It is a oriented time like surface in spacetime. Now the shape functions become time-independent, $X^\mu = X^\mu(\tau, u^A)$, where τ is a coordinate that labels the leafs of the foliation of m by Σs.

The time evolution of the embedding functions for Σ in to the worldvolume can be written as

$$\dot{X}^\mu = N\eta^\mu + N^A \epsilon^\mu_A, \tag{29}$$

where η^μ is the unit (future-pointing) timelike normal to Σ into m, N is called the lapse functions and N^A the shift vector [9]. We can note that the content of this equation is simply that the time evolution of Σ is into the worldvolume m.

The geometry of the worldvolume can be represented in parametric form by the embedding functions $x^\mu = \chi^\mu(\xi^a)$, where $\xi^a = (\tau, u^A)$ are local coordinates for m (see the paragraph below Eq.(3)), and χ^μ the embedding functions ($a = 0, 1, ..., p$).

The tangent vectors to m, $e_a = e^\mu_a \partial_\mu$, decompose in a part tangential to Σ and part along the time evolution of Σ,

$$e^\mu_a = \begin{pmatrix} \dot{X}^\mu \\ \epsilon^\mu_A \end{pmatrix}. \tag{30}$$

Thus, the worldvolume induced metric, $\gamma_{ab} = e^\mu_a e^\nu_b g_{\mu\nu}$[5, 7], takes the familiar ADM form

$$\gamma_{ab} = \begin{pmatrix} -(N^2 - N^A N^B) h_A B & h_{AB} N^B \\ h_{AB} N^B & h_{AB} \end{pmatrix}. \tag{31}$$

we can see that the worldvolume element is given by $\sqrt{-\gamma} = N\sqrt{h}$. The expression given in (31) is the same found in [9, 12].

With these results, we can calculate the total momentum $P^\mu(\Sigma)$ of the spatial hypersurfaces, this is, taking in account the eqution (14) we find

$$P^\mu(\Sigma) = \int_\Sigma \tau_a P^{a\mu} = -\alpha \int_\Sigma \sqrt{h}\eta^\mu, \tag{32}$$

where $\eta^\mu = \tau^a e^\mu_a$ is the unit velocity vector at a given point on Σ. The momentum density $P^{a\mu}$ is not only tangent to the worldsheet, it also lies parallel to the tangent vector, e^μ_a . In this sense, the extremal surfaces, like geodesics, are self-parallel. It is important to mention, that the momentum (32) is found in [12], however, in [12] the geometric content of (32) is confuse. Thus, with the results of this paper is easier to obtain it and interpret in a geometry context.

On the other hand, using (31) we can write the [DNG] action for bosonic p-branes as $S = \int d\tau L[X, \dot{X}]$, with the lagrangian given by

$$L[X, \dot{X}] = -\alpha \int_\Sigma d^p u \sqrt{-\gamma} = -\alpha \int_\Sigma d^p u N \sqrt{h}, \tag{33}$$

thus, using (32) and (33) we find that

$$H[X, P] = \int d^p u [P_\mu \dot{X}^\mu] - L[X, \dot{X}] = 0, \tag{34}$$

such as expected from worldvolume reparametrization invariance the hamiltonian vanish. In this manner, we can see that the hamiltonian is a linear combination of the phase space constraints that generate reparametrizations $\{F_0, F_A\}$. This constraints, using (32) is geven by

$$F_A = P_\mu \epsilon^\mu_A = 0, \tag{35}$$

$$F_0 = P^2 + \mu^2 h = 0, \tag{36}$$

and the Hamiltonian takes the form

$$H[X, P] = \int_{\Sigma_\tau} [\lambda F + \lambda^A F_A], \tag{37}$$

where λ and λ^A are Lagrange multipliers enforcing the constraints. To find this Lagrange multipliers we calculate the first Hamilton equation, using the Eqs. (35), (36) and (37) we find

$$\dot{X}^\mu = \frac{\delta H}{\delta P_\mu} = 2\lambda P^\mu + \lambda^A \epsilon^\mu_A, \tag{38}$$

from Eqs. (29) and (38) we can identify that; $\lambda = \frac{N}{2\alpha\sqrt{h}}$ and $\lambda^A = N^A$. We can see that all this results was obtained in [9, 12] by a different way.

To finish, is important to mention that the constraint (35) generates diffeomorphisms tangential to Σ; F_0 is a universal constraint for all reparametrizations invariant action, they are the generators of worldvolume reparametrizations. All this symmetries, in the gauge invariance of ω (Eq. (1)) are included; in [5] we prove that ω is invariant under spacetime diffeomorphisms and worldvolume reparametrizations. This is, our simplectic structure ω inherits the covariant properties of the deformation formalism, from which it has been constructed [5].

In addition to this work, we know that Polyakov action of the p-branes is more suitable for quantization, however, with the results of this paper we will see in coming works if there are any progress with [DNG] action, the details of this work with the Polyakov action we will discuss in the future. The reason is that we need to develop a covariant formalism of deformation as in [7] and [18] for the Polyakov action, and to do an analisis like this work for such geometry. On the other hand, important results in this direction can be found in [11, 12].

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